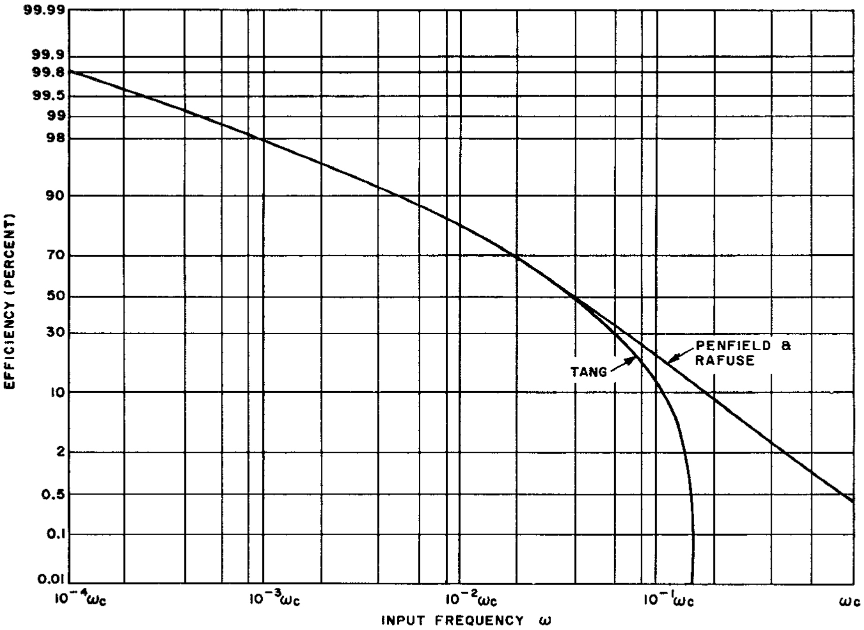

Fig. 3. The efficiency of a doubler as a function of ω.

P_{out} we obtain the value $S_{\max}$ of the sought-after varactor as:

$$S_{\max} = 0.08504f(V_B + \phi)[(V_B + \phi) \pm \sqrt{(V_B + \phi)^2 - 984.27R_sP_{\text{out}}}]/P_{\text{out}}. \quad (28)$$

The other sought-after quantities are:

$$R_l = 0.02389\left(\frac{S_{\max}}{f}\right) - R_s \quad (29)$$

$$R_{\text{in}} = 0.011945\left(\frac{S_{\max}}{f}\right) + R_s = \frac{R_l}{2\epsilon} \quad (30)$$

$$L_l(\text{load inductance}) = 0.0031662\,\frac{S_{\max}}{f^2} \quad (31)$$

$$L_0(\text{source impedance required}) = 4L_l \quad (32)$$

$$V_b(\text{bias voltage}) = 0.3596(V_B + \phi) - \phi, \quad (33)$$

and

$$P_d = 19.9327R_sf^2(V_B + \phi)^2/S_{\max}^2. \quad (34)$$

In passing, we shall supply the simple proof that except for the doubler, any abrupt-junction diode frequency multiplier without an idler is not possible. Let the current and charge flowing through the diode be

$$i = i_1 + i_n = I_1 \cos \omega t + I_n \cos (n\omega t + \theta), \quad (35)$$

and

$$q = q_1 + q_n = Q_1 \sin \omega t + Q_n \sin (n\omega t + \theta) + K. \quad (36)$$

Using (4), we have the voltage across the diode

$$v = -\phi + \frac{S_{\max}^2}{4(V_B + \phi)} \cdot \Big\{\Big[\frac{1}{2}(Q_1^2 + Q_n^2) + (K + Q_\phi)^2\Big] + 2Q_1(K + Q_\phi)\sin \omega t - \frac{Q_1^2}{2}\cos 2\omega t + Q_1Q_n \cos[(n-1)\omega t + \theta] + 2Q_n(K + Q_\phi)\sin(n\omega t + \theta) - Q_1Q_n\cos[(n+1)\omega t + \theta] - \frac{Q_n^2}{2}\cos(2n\omega t + 2\theta)\Big\}. \quad (37)$$

Comparing (35) with (37), we notice that except for $n=2$ case the fundamental voltage is always in quadrature with the fundamental current and, likewise, nth harmonic voltage is always in quadrature with nth harmonic current.

On the other hand, the same approach can be used for frequency multipliers other than the abrupt-junction doubler by adding the necessary idler current or currents flowing through the varactor as constraints.

It can be shown that doublers are possible for hyperabrupt junctions with γ(doping profile exponent) $= \frac{2}{3}$ and quadruplers without idler are possible for the hyperabrupt functions with $\gamma = \frac{3}{4}$.

The results obtained by the present approach are quite different from those of the Penfield-Rafuse approach using Fourier expansion of nonlinear elastance. The present approach emphasizing minimum dissipation can be particularly useful in cases where dissipation is the principle problem; for example, for high power varactor multipliers, minimum dissipation is often the design objective, not maximum efficiency or maximum power output.

The present approach is simple and straightforward in concept and does not require the aid of a computer.

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Optimum Pitch of Traveling-Wave Masers

The purpose of this correspondence is to show theoretically that there exists an optimum pitch (p in Fig. 1), which gives maximum net gain for traveling-wave masers (TWM) [1], utilizing transverse strip slow wave structures (e.g., comb-structures [2], Karp-structures [3], and meander lines). The net gain in dB of the traveling-wave maser may be expressed [1] as

$$G = 27\frac{L}{\lambda_0}s\left[\frac{1}{|Q_m|} - \frac{1}{Q_0} - \frac{1}{Q_i}\right] \quad (1)$$

where L is the structure length, λ_0 the free space wavelength, s the slowing factor, $|Q_m|$ the magnetic quality factor of the maser material, Q_0 the ohmic Q-factor, and Q_i the Q-factor related to the forward wave losses in the isolator. The Q-factors depend on the particular structure geometry.

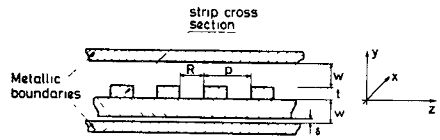

Fig. 1. A typical cross section of transverse strip slow-wave structures.

A typical cross section of a transverse strip slow-wave structure is shown in Fig. 1. If the cross-sectional dimensions within one period p are much smaller than the free space wavelength, and if the strip lengths are long compared to their cross sections, the fields may be approximated by TEM waves traveling along the strips (the x direction of Fig. 1).

Assume the pitch of a particular slow-wave structure is p_0. The slowing of the structure becomes s_0 and the ohmic Q-factor Q_{00}. A scaling of the cross section is obtained when all cross-sectional dimensions are multiplied by the factor p/p_0. Hence, the new pitch becomes p.

The impedance of the TEM waves $K(\phi)$ is only dependent on the relative cross-sectional dimensions and, consequently, remains constant during the scaling procedure. $K(\phi)$ does, however, depend on the phase shift ϕ between two strips [2], [3]. If the propagation constant of the wave traveling along the structure (the z direction of Fig. 1) is β, we have

$$\phi = \beta \cdot p.$$

The $\omega - \phi$ characteristic of the structure is determined by the boundary conditions of the TEM waves, the length of the strips, and the impedance $K(\phi)$ [2], [3]. We now assume that the boundary conditions, the length of the strips, and the cross-sectional dimensions divided by p is constant during a scaling of the cross section by p/p_0. Hence, the $\omega - \phi$ characteristic is independent of p. The group velocity v_g and the slowing s of the wave traveling in the z direction becomes

$$v_g = \frac{d\omega}{d\beta} = p\frac{d\omega}{d\phi}$$

$$s = \frac{c_0}{v_g} = \frac{c_0}{p_0 \cdot \frac{d\omega}{d\phi}}\frac{p_0}{p} = s_0 \cdot \frac{p_0}{p} \quad (2)$$

where c_0 is the velocity of light in vacuum.

The ohmic quality factor Q_0 is proportional to V/S for constant relative cross-sectional dimensions, where V is the volume and S is the surface of one period of the struc-

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ture. One obtains

$$Q_0 = Q_{00} \cdot \frac{p}{p_0} \cdot \tag{3}$$

The magnetic quality factor Q_m is independent of p for constant relative cross-sectional dimensions. Since the isolator Q-factor Q_i depends on the fields in the slow wave structure in the same way as Q_m [4], we can write

$$\frac{1}{Q_i} = r \cdot \left| \frac{1}{Q_m} \right| \tag{4}$$

where $r<1$ and typically $r<0.2$. Using (1) through (4), we now obtain the net gain

$$G(p) = G(p_0) \frac{\dfrac{p_0}{p} - \dfrac{|Q_m|}{(1-r)Q_{00}}\left(\dfrac{p_0}{p}\right)^2}{1 - \dfrac{|Q_m|}{(1-r)Q_{00}}} \cdot \tag{5}$$

The net gain $G(p)$ has a maximum for

$$p = p_{\text{opt}} = p_0 \frac{2|Q_m|}{(1-r)Q_{00}} \tag{6}$$

i.e.,

$$Q_{0,\text{opt}} = \frac{1-r}{2|Q_m|} \cdot \tag{7}$$

The optimum net gain becomes

$$\begin{aligned} G(p_{\text{opt}}) &= G(p_0) \frac{p_0/p_{\text{opt}}}{2 - p_{\text{opt}}/p_0} \\ &= 27 \frac{L \cdot s}{\lambda_0} \frac{1-r}{2|Q_m|} \cdot \end{aligned} \tag{8}$$

In Fig. 2 $G(p_{\text{opt}})/G(p_0)$ is plotted for different values of p_0/p_{opt}. The optimum pitch represents a compromise between the ohmic losses and the slowing factor, and the optimum gain is obtained when the electronic gain (the gain obtained for $Q_i = Q_0 = \infty$) is approximately twice the ohmic losses (7).

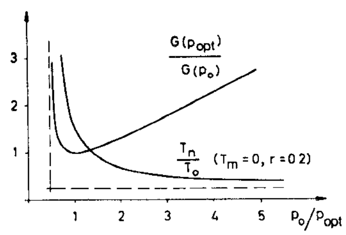

Fig. 2. $G(p_{\text{opt}})/G(p_0)$ with G in dB and T_n/T_0 with $T_m=0$ and $r=0.2$ as a function of p_0/p_{opt}.

For $p_0 \leq \frac{1}{2} p_{\text{opt}}$ the ohmic losses becomes so high that no net gain is obtainable. It may be advisable to choose $1 \leq p_0/p_{\text{opt}} \leq 1.5$.

The noise temperature for the pitch p_0 of the TWM [1] becomes, using (3) and (7):

$$\begin{aligned} T_n &= \frac{G-1}{G} \frac{1/|Q_m|}{1/|Q_m| - 1/Q_i - 1/Q_{00}} T_m \\ &\quad + \frac{1/Q_i + 1/Q_{00}}{1/|Q_m| - 1/Q_i - 1/Q_{00}} T_0 \\ &= \frac{G-1}{G} \left[\frac{p_0/p_{\text{opt}}}{2p_0/p_{\text{opt}} - 1} \cdot \frac{2T_m}{1-r} \right. \\ &\quad \left. + \frac{r + 1/(2p_0/p_{\text{opt}} - 1)}{1-r} T_0 \right]. \end{aligned} \tag{9}$$

For $p_0 = p_{\text{opt}}$, $r=0.2$, $T_m \ll T_0$ one obtains $T_n = 1.5 \cdot T_0$, a reasonable noise temperature for liquid helium cooled masers. For high temperature masers it may be wiser to choose a pitch larger than the optimum pitch in order to obtain a lower noise temperature (see Fig. 2).

It may be emphasized that the scaling procedure proposed here does not change the frequency characteristic of the pass band, i.e., the tunable bandwidth of the maser remains constant. Further, the coupling to the structure is also unchanged since the impedance levels are constant. The saturation power at the signal frequency of the maser is changed with a factor of p/p_0.

Investigations of published data of several traveling-wave masers reveal that the net gain could have been considerably increased by optimizing the pitch. Very often the optimum pitch is shorter than the design pitch by a factor of 2 to 3, leading to a loss in net gain (L unchanged) by a factor of 1.33 to 1.80. A substantial decrease of the pitch and a scaling down of the cross-sectional dimensions correspondingly may, of course, in some cases lead to fabrication difficulties.

Care has to be taken in applying the foregoing results to longitudinally stagger-tuned masers, since $|Q_m|$ in that case becomes different in different sections of the TWM. If the stagger-tuning is performed transversely, i.e., the paramagnetic line is broadened identically for each period, the results are very well applicable

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Measurement of Microwave Power in WR137 Waveguide (5.85–8.20 GHz)

The Radio Standards Laboratory of the NBS Institute for Basic Standards (U. S. Department of Commerce), Boulder, Colo., announces a calibration service in WR137 waveguide for the measurement of effective efficiency, efficiency, and calibration factor of bolometer units, and of calibration factor of bolometer-coupler units. The measurement of efficiency of a bolometer unit is made available for the first time by the Radio Standards Laboratory, but for the present is limited to WR137 waveguide. Although calibrations can be performed within the useful range of the waveguide, some degree of economy to the customer results if calibrations are performed at the selected[1] frequencies of 6.45, 7.00, and 7.40 GHz.

The four quantities measured are defined as follows:

Effective Efficiency for Bolometer Units: The ratio of the substituted dc power in the bolometer unit to the microwave power dissipated within the bolometer unit.

Calibration Factor for Bolometer Units: The ratio of the substituted dc power in the bolometer unit to the microwave power incident upon the bolometer unit.

Efficiency for Bolometer Units: The ratio of the microwave power absorbed by the barretter element to the microwave power dissipated within the bolometer unit.

Calibration Factor for Bolometer-Coupler Units: The ratio of the substituted dc power in the bolometer unit on the side arm of the directional coupler to the microwave power incident upon a nonreflecting load attached to the output port of the main arm.

Because of certain limitations in adapting the microwave microcalorimeter measurement technique to the larger waveguide sizes, the effective efficiency and calibration factor of bolometer units in WR137 waveguide are measured somewhat indirectly by using a working standard that has been calibrated by the impedance method.[2–4] The impedance technique yields a direct measurement of the efficiency of a bolometer unit. This efficiency measurement, usually of special interest only, is available for the first time at the Radio Standards Laboratory as a calibration service. The bolometer units must be fitted with a barretter element of a purely resistive type (thermistor-type element cannot be used) with a nominal resistance of 200 ohms. The efficiency measurements are performed with an uncertainty no greater than ± 1 percent.

The effective efficiency and calibration factor of a bolometer unit, as well as the calibration factor of a bolometer-coupler unit, are measured with an uncertainty no greater than 2 percent in WR137 waveguide. For these measurements the element can be of the barretter or thermistor type, and of either 100- or 200-ohm resistance, operating at a bias current between 3.5 and 15 mA. The bolometer unit can be of either the fixed-tuned or untuned broadband type. Power measurements can be made on bolometer units over a range of 0.1 to 10 mW.

Power measurements can be made on bolometer-coupler units in WR137 waveguide with coupling ratios from 3 to 20 dB. A bolometer unit of either the fixed-tuned or untuned broadband type must be permanently attached to the side arm of the coupler. The coupler should have a directivity of no less than 40 dB, and a VSWR no greater than 1.05 for the input and output ports of the main arm of the coupler.

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[1] Performing microwave calibrations, a considerable amount of time is usually needed to prepare the system for a measurement operation. Much of this preparation is related to adjustment of the system to the frequency of operation selected for the calibration. Time and cost often can be reduced by minimizing the number of times the operating frequency of the calibration system must be readjusted.

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